ISI Type-A Mock Test

Circle the correct option. Correct Answer = 4 marks, Leave Blank = 1, Wrong Answer = 0

Name:
Date:

1. All 2 digits number from 19 to 93 are written in an arbitrary order to form a number, for example $N = 192021\ldots93$ can be one such number. The largest power of 3 that divides such a number is;
A. $3^0 = 1$ B. $3^1 = 3$ C. $3^2 = 9$ D. $3^4 = 81$

2. For two coprime positive integers m and n both being more than 1, $\log_{10} m/\log_{10} n$ is;
A. is always rational number. B. can sometimes be rational number. C. is always an irrational number. D. is always a root of some polynomial function.

3. The number of primes p such that there exists two integers x and y satisfying $(p+1) = 2x^2$ and $(p^2+1) = 2y^2$ is; A. None. B. Only one. C. More than one but finitely many. D. Infinitely many.

4. Let b be a non-zero real number. Suppose that the quadratic equation $2x^2 + bx + \frac{1}{b} = 0$ has two distinct real roots. Then; A. $b^2 - 3b > -2$. B. $b + \frac{1}{b} > \frac{5}{2}$ C. $b + \frac{1}{b} < \frac{5}{2}$ D. $b^2 + \frac{1}{b^2} < 4$

5. Let P be an interior point of a convex quadrilateral ABCD, and let K, L, M, N be midpoints of AB, BC, CD, DA respectively. If Area(PKAN) = 25, Area(PLBK) = 36 and Area(PMDN) = 41, then Area(PLCM) is; A. 20 B. 29 C. 52 D. 54

6. The largest number in the sequence $\sqrt[1]{1}, \sqrt[2]{2}, \sqrt[3]{3}, \ldots \sqrt[n]{n}, \ldots$ is; A. 1. B. $\sqrt[2]{2}$ C. $\sqrt[3]{3}$ D. $\sqrt[8]{8}$

7. If $a_0, a_1, \ldots$ be the coefficients of the polynomial $(1+x+x^2)^{2019}$, starting with the constant term being a_0, then the sum $S = a_0 + a_2 + a_4 \ldots$ is;
A. is prime. B. is divisible by 2019. C. is odd. D. is even.

8. The roots of the polynomial $p(x) = x^6 + ax^3 + bx^2 + cx + d$ where a, b, c, d are real numbers, are; A. are all real for any choice of a, b, c, d. B. are not all real for any choice of a, b, c, d. C. are all real for some choice of a, b, c, d. D. None of these.

9. The locus of the points (x, y, z) such that, $x^2 + y^2 + z^2 = 6$ and $x + y + z = 4$ is a / an; A. point. B. plane. C. ellipse. D. circle.

10. Consider the quadratic polynomial $p(x) = x^2 + 2019x - 12345$. For how many positive integers n, there exists another integer M such that $p(n)p(n+1) = p(M)$? A. None. B. Less than 2020 C. More than 2020 but finitely many. D. Infinitely many.

11. Let, $A_1 = \left\{2(-1)^{n+1} + (-1)^{n(n+1)/2}\left(2 + \frac{3}{n}\right), n \in \mathbb{N}\right\}$ and $A_2 = \left\{\frac{n-1}{n+1}\cos\frac{2n\pi}{3}, n \in \mathbb{N}\right\}$, then; A. $\sup A_1 > \sup A_2$ and $\inf A_1 < \inf A_2$. B. $\sup A_1 > \sup A_2$ and $\inf A_1 > \inf A_2$. C. $\sup A_1 < \sup A_2$ and $\inf A_1 < \inf A_2$. D. $\sup A_1 < \sup A_2$ and $\inf A_1 > \inf A_2$.

12. $\lim_{x\to 0}\left(x^2\left(1 + 2 + 3 + \cdots + \left[\frac{1}{|x|}\right]\right)\right)$; A. Does not exist. B. 0 C. 1/2 D. 1

13. Consider a function;

$$f(x) = \begin{cases} |x| & \text{when x is irrational or x = 0} \\ 1/q & \text{when } x = p/q, \text{ with } p, q \in \mathbb{Q} \end{cases}$$

Then f is continuous at; A. Nowhere. B. Only $x = 0$. C. At any irrational number and at 0. D. Everywhere.

14. **One SAQ worth 12 marks (4 marks bonus)**
    (a) Does there exists a bounded function $f(\cdot)$ defined on $[0, 1]$ such that it neither attends its infimum nor supremum? If yes, give a concrete proof. If no, give a counterexample.
    (b) Give an example of a function $f(\cdot)$ defined on $[0, 1]$ such that, it does not achieve its infimum on any $[a, b] \subsetneq [0, 1]$, i.e. $[a, b]$, where $a < b$ is a strict subset of $[0, 1]$.